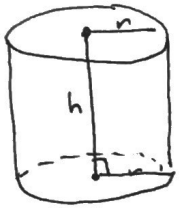
12.2 - Circular Solids & Surface Area

- We can make 3d shapes from circles similarly to how we formed polyhedra from polygons.
- 3 main types: cylinders, cones, spheres.

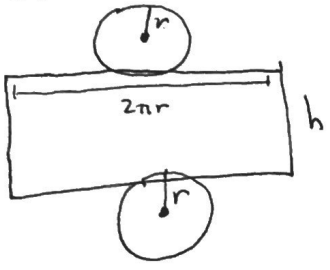
Cylinders

- nice cylinders:



right circular cylinder: have two bases that are circles of radius r, height h.

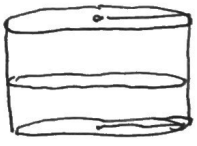
Net:



SA: $2\pi rh + 2\pi r^2$ for right circular cylinders only.
(rect. / 2 circles)

Other cylinders:

Can have non-circular bases:



elliptical cylinder

Non-right cylinder (oblique)

- no nice formula for surface area.

- think that cylinders are like prisms

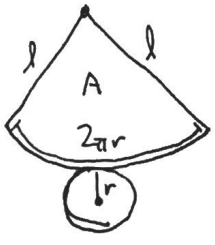
Cones

- like pyramids

right circular cones: base is a ~~circle~~ circle, height from top of cone meets the center of the circle at 90° angle.
- measure slant height, ℓ, as for pyramids

ex:

Net:



What is the ~~area~~ SA?

small circle: πr^2

big sector: the ratio of the sector's arc length to its full circle is $\frac{2\pi r}{2\pi \ell} = \frac{r}{\ell}$ so this is also the ratio of the areas:

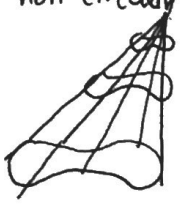
$$\frac{A}{\pi \ell^2} = \frac{r}{\ell} \Rightarrow A = \pi r \ell.$$

So the total surface area is $\boxed{\pi r \ell + \pi r^2}$

Other cones:

ex: non right => oblique

ex: non-circular base



Spheres

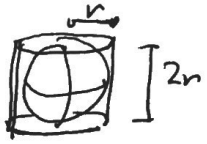
- ball in 3-d space
- set of points in space a fixed distance r (radius) away from a given point (center).
- cross-sections through the center are circles of radius r.

Surface area of a sphere: $\boxed{SA = 4\pi r^2}$

- 4 times the area of a cross section.

Why?

1) Try it. e.g. peel an orange and arrange the pieces to fill 4 circles of the same radius of an orange.



2) Archimedes: a) Put a cylinder around the sphere.

b) Project the sphere horizontally onto the wall of the cylinder (imagine poking a hole at the top and pushing the sphere out onto the cylinder).

c) This preserves area. So SA of sphere = $2r \cdot (2\pi r) = 4\pi r^2$.

3) Calculus.